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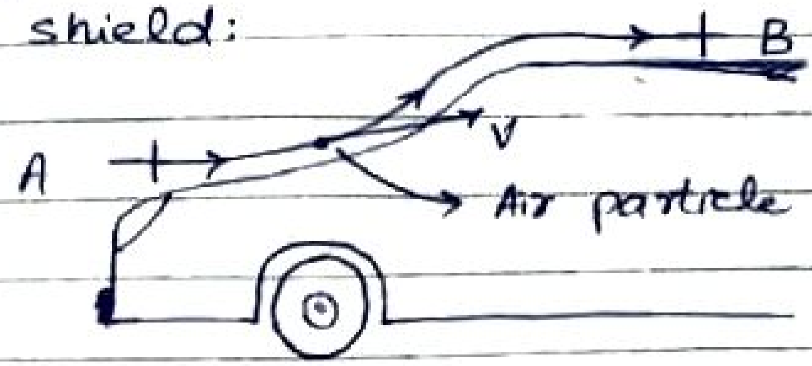
MECH-324
Fluid Mechanics-I
Homework-2

(1) Consider the flow of an air particle along the streamline AB over the wind shield:

Diagram



The air particle travels in a stream line is immersed in the pressure field produced by the surrounding air particles. In this streamline, the gravity and the pressure affects are balanced by the centrifugal acceleration such that:

$$-\gamma \frac{\partial z}{\partial n} - \frac{\partial p}{\partial n} = \rho \frac{V^2}{R}$$

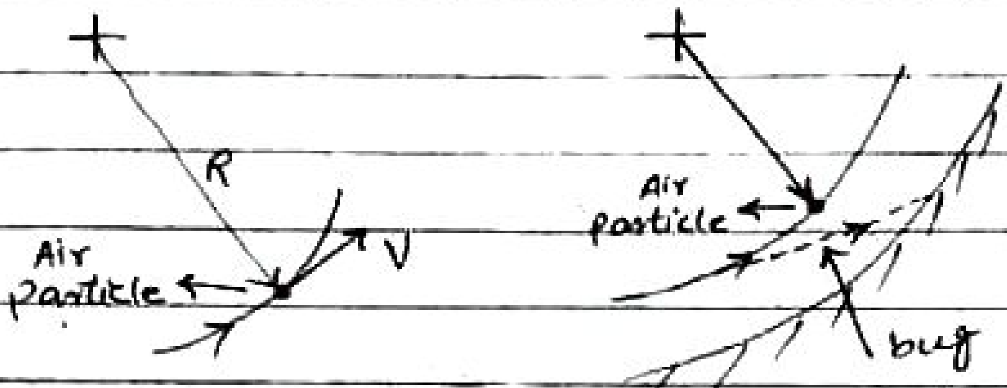
But, a bug in air has more density than the density of air, and it experiences the same pressure field as air particles. which is not sufficient to make it turn as the air particle does. Mathematically,

$$\rho_{bug} > \rho_{air}$$

To balance the forces,

$R_{bug} > R_{air}$ if $\rho_{bug} > \rho_{air}$

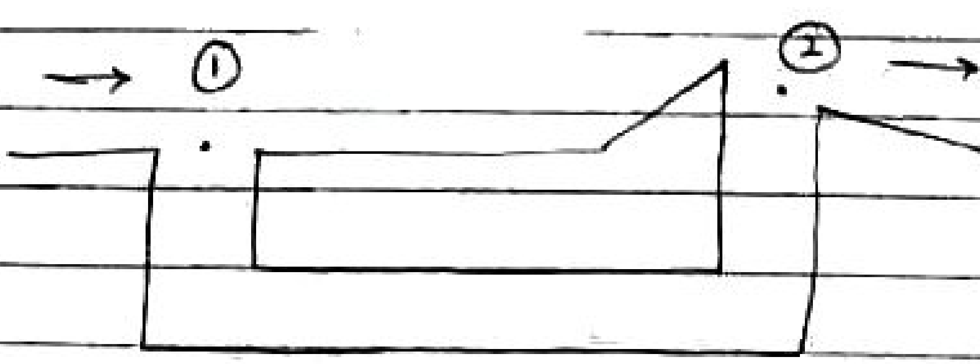
Therefore, the bug hits the wind shield. The flow patterns for the air and bug are shown in the following figure:



Flow of air particle. Flow of a bug.

(2) Given data:

$V_0 = 6 \text{ m/s}$ $\quad V_2 = 1.07\, V_0$



Bernoulli's equation:

$$P_1 + \frac{\rho V_1^2}{2} + \gamma z_1 = P_2 + \frac{\rho V_2^2}{2} + \gamma z_2 \rightarrow (1)$$

Here $z_1 = z_2$

Now eq. (1) becomes:

Substituting values:

$$P_1 + \frac{1}{2}\rho V_0^2 = P_2 + \frac{1}{2}\rho V_0^2 \times 1.07^2$$

$$P_1 - P_2 = \frac{1}{2}\rho V_0^2 [1.07^2 - 1]$$

$$P_1 - P_2 = \frac{1}{2} \times 1.23 \times 6^2 \times 0.1449$$

$$\boxed{P_1 - P_2 = 3.208\ N/m^2} = \text{Ans}$$

③ Calculating fluid velocity:

$$P_0 = P_S + \frac{1}{2}\rho V^2$$

$$V_{fluid} = \sqrt{\frac{2(P_0 - P_S)}{\rho_{fluid}}} \rightarrow ①$$

Pressure difference:

$$(P_0 - P_S) = (\gamma_{mano} - \gamma_{fluid}) \times h$$

Here $\gamma_{mano} >> \gamma_{fluid}$

$$(P_0 - P_s) = \gamma_{mano} \times h \rightarrow ②$$

$$\rho_{fluid} = P/RT \rightarrow ③$$

Substituting eq. ②, ③ in eq. ①

$$V_{fluid} = \sqrt{\frac{2\,\gamma_{mano} \times h}{P/RT}}$$

Substituting values.

$$V_{fluid} = \sqrt{\frac{2 \times 62.4 \times 2.3}{25 \times 144 / 1.242 \times 10^4 \times (40+460)}}$$

$$\boxed{V_{fluid} = 203 \text{ ft/s.}} = \underline{\text{Ans.}}$$

Calculating Mach number:

$$Ma = V/C = V/\sqrt{KRT}$$

$$Ma = 203/\sqrt{1.66 \times 1.242 \times 10^4 \times 500}$$

$$Ma = 0.0632 < 0.3$$

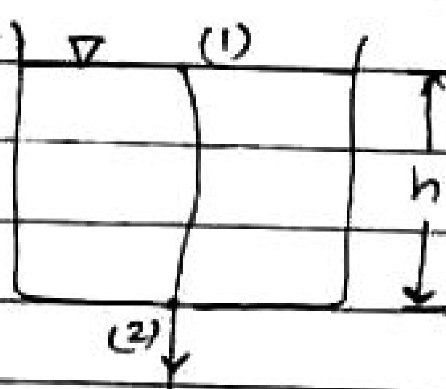
Flow will be incompressible.

(4) Flow velocity of water $= V_2 = 8$ m/s

Applying Bernoulli's eq.

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Here:



$P_1 = P_2 = 0$

$V_1 = 0$

$z_2 = 0$

$z_1 = h$

Now, $\gamma h = \frac{1}{2}\rho V_2^2 \Rightarrow \rho g h = \frac{1}{2}\rho V_2^2$

$$h = V_2^2/2g = (8)^2/2 \times 9.81$$

$$\boxed{h = 3.26 \text{ m}} = \underline{\text{Ans.}}$$

⑤ ~~Applying Bernoulli's equation:~~

$$P_A + \frac{\rho V_A^2}{2} + \gamma Z_A = P_2 + \frac{\rho V_2^2}{2} + \gamma Z_2$$

$$P_A + 0 + \gamma(20) = 0 + 0 + \gamma(h+20)$$

$$P_A = \gamma h$$

Applying Bernoulli's equation:

$$P_A + \frac{\rho V_A^2}{2} + \gamma Z_A = P_1 + \frac{\rho V_1^2}{2} + \gamma Z_1$$

$$\gamma h + 0 + \gamma(20) = (25 \times 144) + 0 + \gamma(8)$$

$$62.4(h) + 62.4(20) = (25 \times 144) + 62.4(8)$$

$$\boxed{h = 45.7 \text{ ft}} = \text{Ans}$$

⑥ (a) Calculating the distance from the origin as follows:

$$\rho = \sqrt{x^2 + y^2}$$

Fluid flow speed:

$$V = \sqrt{u^2 + v^2}$$

Given that:

$$u = \frac{-ky}{x^2+y^2} \quad ; \quad v = \frac{kx}{x^2+y^2}$$

$$V = \sqrt{\left[\frac{-ky}{x^2+y^2}\right]^2 + \left[\frac{kx}{x^2+y^2}\right]^2}$$

$$V = \sqrt{\frac{k^2(x^2+y^2)}{(x^2+y^2)^2}} = \frac{k}{\sqrt{x^2+y^2}}$$

$$V = k/r$$

Therefore, field velocity is inversely proportional to distance from origin.

⑥ (b) Streamline equation:

$$\frac{dy}{dx} = \frac{v}{u}$$

Substituting v and u value:

$$\frac{dy}{dx} = \frac{\frac{kx}{x^2+y^2}}{\frac{-ky}{x^2+y^2}} = \frac{-x}{y}$$

$$-x\,dx = y\,dy$$

Integrating on both sides.

$$\int -x\,dx = \int y\,dy$$

$$\frac{-x^2}{2} = \frac{y^2}{2} + C_1$$

$$-x^2 = y^2 + 2C_1$$

$$x^2 + y^2 = -2C_1$$

$$x^2 + y^2 = C$$

It is an equation of circle, hence it forms a circle.

(7) Given velocity equation:

$$V = V_0\left[1 - e^{-ct}\right]\left[1 - \frac{x}{l}\right]$$

Equation for x-component of acceleration:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

Substituting value of "V":

$$a_x = \left\{\left[\frac{\partial}{\partial t}\left(V_0\left(1-e^{-ct}\right)\left(1-\frac{x}{l}\right)\right)\right] + \left[V_0\left(1-e^{-ct}\right)\left(1-\frac{x}{l}\right)\frac{\partial}{\partial x}\left(V_0\left(1-e^{-ct}\right)\left(1-\frac{x}{l}\right)\right)\right]\right\}$$

$$a_x = \left\{\left[\text{[redacted]}\; V_0\left(+e^{-ct}\right)c\left(1-\frac{x}{l}\right)\right] + \left[V_0\left(1-e^{-ct}\right)\left(1-\frac{x}{l}\right)\left(V_0\left(1-e^{-ct}\right)\left(\frac{-1}{l}\right)\right)\right]\right\}$$

$$a_x = \left\{\left[V_0\left(1-\frac{x}{l}\right)ce^{-ct}\right] + \left[V_0^2\left(1-e^{-ct}\right)^2\left(1-\frac{x}{l}\right)\left(-1/l\right)\right]\right\}$$

$$a_x = V_0\left(1-\frac{x}{l}\right)\left[ce^{-ct} - \frac{V_0}{l}\left(1-e^{-ct}\right)^2\right]$$

This equation gives acceleration as a function of x and t.

$$a_x = V_0\left(1-\frac{x}{\ell}\right)\left[ce^{-ct} - \frac{V_0}{\ell}\left(1-e^{-ct}\right)^2\right]$$

$$0 = 10\left(1-\frac{x}{\ell}\right)\left[ce^{-ct} - \frac{10}{5}\left(1-e^{-ct}\right)^2\right]$$

For any value of x and $t = 1s$

$$\left[ce^{-ct} - 2\left(1-e^{-ct}\right)^2\right] = 0$$

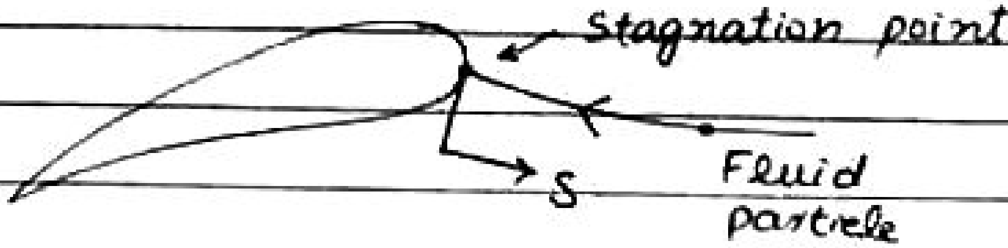
$$ce^{-ct} = 2\left(1-e^{-ct}\right)^2$$

Solution of the root:

$$\boxed{c = 0.49\ s^{-1}}$$ Ans.

In this problem, the local acceleration ($\partial u/\partial t > 0$) is balanced by the convection deceleration ($u\frac{\partial u}{\partial x} < 0$). If the flow rate increases with time, fluid flows to the area of least velocity.

(8) (a)



$s = 0.6$ ft ; $s = 0.6e^{-0.5t}$

Velocity of particle:

$$V_{particle} = \frac{ds}{dt} = \frac{d}{dt}\left(0.6e^{-0.5t}\right)$$

$$V_{particle} = (0.6)(-0.5)\,e^{-0.5t}$$

$$\boxed{V_{particle} = -0.3\,e^{-0.5t} \quad ft/s} = Ans.$$

(b) From the above eq.

$$V_{particle} = (-0.5)(0.6e^{-0.5t})$$

$$\boxed{V_{particle} = -0.5\,s \quad ft/s} = Ans.$$

(c) Fluid acceleration:

$$a_s = V\frac{dV}{ds}$$

$$a_s = (-0.5s)\left[\frac{d}{ds}(-0.5s)\right]$$

$$a_s = (-0.5s)(-0.5)$$

$$\boxed{a_s = 0.25\,s \quad ft/s^2} = Ans.$$

(9) Normal acceleration:

$$a_n = V^2/R = V^2/\infty$$

$$a_n = 0$$

Tangential acceleration:

$$a_s = -V\frac{dV}{dr}$$

Here $V = V_0 R^2/r^2$

$$a_s = -\frac{V_0 R^2}{r^2}\cdot\frac{d}{dr}\left[\frac{V_0 R^2}{r^2}\right]$$

$$a_s = -\frac{V_0 R^2}{r^2}\left[\frac{-2V_0R^2}{r^3}\right]$$

$$a_s = \frac{2V_0^2R^4}{r^5}$$

Here $V_0 = 5$ ft/s $\quad R = 2$ ft

$$a_s = \frac{2(5)^2 \times (2)^4}{r^5} = \frac{800}{r^5}$$

At $r = 0.5$ ft

$$a_s = \frac{800}{(0.5)^5} = \frac{800}{0.03125}$$

$$\boxed{a_s = 25{,}600 \text{ ft/s}^2} = \text{Ans.}$$

At $r = 2$ ft

$$a_s = \frac{800}{(2)^5} = \frac{800}{32}$$

$$\boxed{a_s = 25 \text{ ft/s}^2} = \text{Ans.}$$

(10) (a) Net flow rate out of system across section CD is:

$$B_{out} = \int_{cs\,out} \rho b \vec{V}.\hat{n}.dA$$

Here $b = 1$; $V.\hat{n} = V\cos\theta$

$$B_{out} = \int_{CD} \rho V\cos\theta . dA$$

$$B_{out} = \rho V\cos\theta \int_{CD} dA$$

$$B_{out} = \rho V\cos\theta\, A_{CD}$$

Here $A_{CD} = l(2\,m)$

$$= \left(\frac{0.5\,m}{\cos\theta}\right)(2\,m)$$

$$= \frac{1}{\cos\theta}\,m^2$$

$V = 3\,m/s$

$$B_{out} = 3\cos\theta \cdot \frac{1}{\cos\theta} \cdot 999$$

$$\boxed{B_{out} = 3000\ kg/s}$$

(b) Net flow rate out of the system across section CD:

$$B_{out} = \int_{cs\ out} \rho b \vec{V} \cdot \hat{n} \cdot dA$$

Here $b = 1/\rho$

$$B_{out} = \int_{CD} V \cdot \hat{n} \cdot dA$$

$$B_{out} = \int_{CD} V\cos\theta\, dA$$

$$B_{out} = V\cos\theta\, A_{CD}$$

$$B_{out} = 3 \times \cos\theta \times \frac{1}{\cos\theta}$$

$$\boxed{B_{out} = 3\ m^3/s} = \text{Ans.}$$